A geometric–algebraic structure underlying prime numbers

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Abstract

We introduce a geometric encoding of divisibility relations on the natural numbers and show that classical primality admits an equivalent structural characterization within this framework. By combining a finite divisibility lattice with elementary congruence constraints, primes are identified as isolated positions free of structural intersections. The approach reformulates primality and factorization as geometric properties and provides a finite, scale-invariant description of the set of prime numbers.

1. Introduction

Let $\mathbb{N} = \{1,2,3,\ldots\}$.
The arithmetic of $\mathbb{N}$is classically studied through algebraic, analytic, and algorithmic methods. In this work we consider an alternative but equivalent representation: divisibility relations are encoded geometrically, and primality is expressed as a structural property of this encoding.

The purpose of the paper is not to derive new analytic estimates, but to provide a rigorous geometric–algebraic reformulation of primality that is finite, explicit, and scale invariant.

2. The divisibility lattice

Definition 2.1

Define the **divisibility lattice**

$$\mathcal{D} := \{(i,j) \in \mathbb{N}^2 : i \mid j\}.$$

We interpret:

- each fixed $i$as generating a periodic structure $\{(i,ki) : k \in \mathbb{N}\}$;
- each fixed $j$as encoding the divisors of j.

Lemma 2.2

For every factorization $j = ab$, the point (a, j)lies in $\mathcal{D}$.

Conversely, each point $(i,j) \in \mathcal{D}$corresponds to a factorization $j = i \cdot (j/i)$.

Thus $\mathcal{D}$encodes the complete multiplicative structure of $\mathbb{N}$.

3. Geometric characterization of primes

Definition 3.1

An integer $p > 1$is called **geometrically prime** if the set

$$\{ i \in \mathbb{N} : (i, p) \in \mathcal{D} \}$$

has cardinality exactly two, namely $\{1, p\}$.

Proposition 3.2

An integer is geometrically prime if and only if it is prime in the classical sense.

Proof.

If $p$is prime, its only divisors are $1$and p, hence the column indexed by $p$contains exactly two points.

If $p$is composite, it admits a nontrivial divisor d, and the column contains at least one additional point.

4. Congruence reduction

Proposition 4.1

Every prime $p > 3$satisfies

$$p \equiv \pm 1 (\mathrm{mod} 6).$$

This induces a restriction of admissible columns in $\mathcal{D}$to two congruence classes modulo 6. The reduction is finite and periodic and excludes all other positions independently of any search procedure.

5. Structural intersections and composite numbers

Definition 5.1

Let $i_1, i_2 \in \mathbb{N}$, $i_1 \neq i_2$.
A column $j$is said to admit a **nontrivial intersection** if both $(i_1, j)$and (i_2, j)belong to $\mathcal{D}$with $1 < i_1, i_2 < j$.

Theorem 5.2

An integer $j > 1$is composite if and only if its column admits a nontrivial intersection.

Proof.

A composite number has at least two distinct nontrivial divisors, producing at least two distinct points in its column.

Conversely, the presence of two such points implies the existence of a nontrivial factorization.

Thus primality corresponds precisely to the absence of nontrivial intersections.

6. Factorization

Within $\mathcal{D}$, the factorization of j is given explicitly by the set

$$\{\, i \in \mathbb{N} : (i, j) \in \mathcal{D} \,\}.$$

Prime factors correspond to minimal generators of the column structure. No inversion or iterative procedure is required.

7. Twin primes

Definition 7.1

A pair $(p, p + 2)$ is a **twin prime pair** if both p and $p + 2$ are geometrically prime.

Twin primes correspond to adjacent admissible columns in the congruence classes $\pm 1 (\mathrm{mod} 6)$ that are both free of nontrivial intersections.

8. Scale invariance

Proposition 8.1

For any $N, k \in \mathbb{N}$, the restriction of $\mathcal{D}$ to columns $\leq N$ is structurally equivalent to its restriction to columns $\leq kN$, up to rescaling.

Hence the geometric criteria for primality are invariant under scaling.

9. Finite structural description of primes

Theorem 9.1

The set of prime numbers is completely characterized by:

1. the divisibility lattice $\mathcal{D}$;
2. finite congruence restrictions;
3. the absence of nontrivial intersections.

No limiting or infinite computational process is required for the definition.

10. Concluding remarks

This work provides a geometric–algebraic reformulation of classical primality. While equivalent to standard definitions, the framework emphasizes structure rather than computation and isolates primality as a finite, scale-invariant property of divisibility relations.

References

1. Euclid, *Elements*, Book IX
2. Hardy, G. H.; Wright, E. M., *An Introduction to the Theory of Numbers*
3. Nathanson, M. B., *Elementary Methods in Number Theory*

Binary code prime number generator:

A)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
3	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1
4	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
5	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0
6	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
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27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

B)

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