A translation-invariant geometric model of primality

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Abstract

We introduce a geometric–structural framework on the integers in which divisibility and primality are characterized as positional properties invariant under translation. The model is based on a periodic decomposition of order six and a notion of structural intersection. Prime numbers are identified as isolated positions on two admissible rays, symmetric with respect to any chosen reference point. The construction extends bidirectionally to all integers and provides a finite, scale- and translation-invariant description equivalent to classical primality, without reliance on limiting processes or infinite computation.

1. Introduction

Classical definitions of primality are formulated algebraically or algorithmically. We propose an equivalent formulation in which primality arises as a geometric property of the integers. The key features of the model are periodicity, translation invariance, and structural isolation.

2. Domain and translation invariance

Let $\mathbb{Z}$denote the set of integers, viewed as an infinite geometric lattice.

Axiom 2.1 (Reference invariance)

Fix an arbitrary reference point $r \in \mathbb{Z}$. All structural properties are expressed relative to $r$and are invariant under changes of reference.

Define translated coordinates $n_r = n - r$.

3. Periodic decomposition

Definition 3.1 (Cycle of order six)

Define the congruence map

$$\varphi(n_r) = n_r \, mod \, 6.$$

This partitions $\mathbb{Z}$into six infinite congruence classes relative to r.

Proposition 3.2

For any $n$with $|\, n_r \,| > 3$,

$$\varphi(n_r) \notin \{1,5\} \implies n \text{ is composite.}$$

4. Admissible rays

Definition 4.1

The admissible set is

$$\mathcal{R}_r = \{n \in \mathbb{Z}: | n_r | > 1, \qquad n_r \equiv \pm 1 (\mathrm{mod} 6)\}.$$

This set decomposes into two infinite arithmetic rays.

Proposition 4.2 (Symmetry)

If $n \in \mathcal{R}_r$, then $2r - n \in \mathcal{R}_r$. Primality is invariant under reflection.

5. Structural intersections

Definition 5.1 (Generator cycles)

For $a \neq 0$, define

$$C_a = \{ak + r: k \in \mathbb{Z}\}.$$

Definition 5.2

An integer $n$admits a nontrivial structural intersection if there exist distinct integers $a, b$with

$$1 <| a |, | b | <| n_r |$$

such that $n \in C_a \cap C_b$.

6. Primality

Theorem 6.1

An integer $n \in \mathcal{R}_r$is composite if and only if it admits a nontrivial structural intersection.

Definition 6.2

An integer $p \in \mathcal{R}_r$is geometrically prime if it admits no nontrivial structural intersection.

Theorem 6.3

Geometric primality is equivalent to classical primality.

7. Scale invariance and infinitude

Proposition 7.1

The structural configuration of admissible rays and intersections is invariant under scaling.

Theorem 7.2

The infinitude of prime numbers follows from the infinite extent of admissible rays and the scale invariance of the isolation criterion.

8. Conclusions

The model provides:

1. a translation-invariant formulation of primality;
2. a finite periodic structure governing an infinite set;
3. a geometric interpretation of factorization as structural intersection;
4. a symmetric extension to negative integers;
5. an intrinsic explanation of infinitude without limiting arguments.

This framework offers an alternative structural perspective on classical primality, emphasizing geometry and invariance rather than computation.

POZIȚIE (N)	MĂRIME (|N|)	FAZĂ (MOD 6)	STARE GEOMETRICĂ
-14	14	**4**	**Compus** 2x7
-13	13	**5**	**Prim (poziție primi gemeni)**
-12	12	**0**	**Compus** 2x2x3
-11	11	**1**	**Prim (poziție primi gemeni)**
-10	10	**2**	**Compus** 2x5
-9	9	**3**	**Compus** 3x3
-8	8	**4**	**Compus** 2x2x2
-7	7	**5**	**Prim (poziție primi gemeni)**
-6	6	**0**	**Compus** 2x3
-5	5	**1**	**Prim (poziție primi gemeni)**
-4	4	**2**	**Compus** 2x2
-3	3	**3**	**„Prim”**
-2	2	**4**	**„Prim”**
-1	1	**5**	**Referință (poziție primi gemeni)**
0	0	**0**	**zero**
1	1	**1**	**Referință (poziție primi gemeni)**
2	2	**2**	**„Prim”**
3	3	**3**	**„Prim”**
4	4	**4**	**Compus** 2x2
5	5	**5**	**Prim (poziție primi gemeni)**
6	6	**0**	**Compus** 2x3
7	7	**1**	**Prim (poziție primi gemeni)**