Structural Reformulation of Divisibility

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We reformulate the arithmetic of the natural numbers as a geometric structure generated by positional relations and distance growth, rather than by algebraic operations. The reformulation does not introduce new arithmetic facts; instead, it reorganizes classical divisibility and primality as emergent properties of a discrete geometric construction.

The underlying principle is that arithmetic relations can be encoded as **incidence and alignment phenomena** within a structured spatial arrangement. In this setting, numbers are not treated primarily as quantities, but as **positions** within a recursively generated geometric configuration.

Primitive Structure

The reformulation assumes only the following primitives:

1. a discrete ordered set of positions;
2. a notion of parallelism between lines;
3. a rule of iterative distance increase between consecutive elements.

No arithmetic operations, equations, or numerical comparisons are assumed at the foundational level.

Generation by Distanciation

The structure is generated row by row. The first row consists of equally spaced points on a line. Each subsequent row is constructed on a parallel line, with the sole rule that the distance between consecutive points increases by one fixed geometric unit relative to the previous row.

This rule defines a unique infinite configuration determined entirely by relative displacement. The construction is local and iterative: each point is obtained by a geometric translation from its predecessor.

Structural Encoding of Divisibility

Within the resulting configuration, certain points align vertically across multiple rows. These alignments are not imposed but arise from the cumulative effect of uniform distance increase.

Divisibility is reformulated as follows:

A position in the base row is divisible by a given level if and only if the corresponding points at that level are vertically aligned.

This definition depends exclusively on geometric coincidence and does not invoke multiplication, division, or counting.

Composite Structure and Intersection

Positions exhibiting multiple vertical alignments correspond to intersections of independent geometric progressions generated by the distanciation rule. Such intersections encode composite structure.

Factorization is thus reformulated as the decomposition of a position into the set of rows with which it aligns. This description is structural and set-theoretic rather than algebraic.

Structural Isolation and Primality

A position is structurally isolated if it aligns only with the trivial levels of the construction. Structural isolation coincides exactly with classical primality.

Thus, primes are identified not by testing divisibility, but by the absence of nontrivial geometric intersections.

Scale and Translation Invariance

The entire construction is invariant under:

- translation of the reference position;
- extension to arbitrarily many rows;
- restriction to finite subconfigurations.

Consequently, no specific origin or numerical scale plays a distinguished role. Arithmetic structure is intrinsic to the configuration.

Equivalence to Classical Arithmetic

The reformulation is equivalent to classical number theory in the following precise sense:

- every classical divisibility relation corresponds to a unique structural alignment;
- every structural alignment corresponds to a classical divisibility relation.

No new arithmetic statements are introduced. The novelty lies exclusively in the structural organization.

Scope of the Reformulation

This work does not propose new computational methods, estimates, or analytic results. It provides a geometric re-encoding of known arithmetic facts, emphasizing structure, invariance, and emergence rather than calculation.

Novelty and Significance of the Geometric Reformulation

The present work introduces a structural viewpoint in which each integer is treated simultaneously as a **value** and as a **position** within a geometric configuration. This dual interpretation is implicit in classical arithmetic but is here made explicit and foundational.

The central novelty lies in the observation that a minimal geometric framework—generated solely by iterative distanciation—suffices to encode the full divisibility structure of the integers. In this framework, primality does not arise from numerical testing, but from **structural isolation** within a globally invariant configuration.

Numbers as values and positions

In standard arithmetic, numbers are primarily regarded as magnitudes. In the present approach, each integer occupies a definite position in a geometric lattice whose organization determines its arithmetic properties.

This positional interpretation is not auxiliary: divisibility, factorization, and primality emerge directly from relative placement. The arithmetic value of a number and its geometric position become inseparable aspects of a single structure.

Geometric generation of divisibility

A key contribution of this work is the demonstration that the divisibility relations among natural numbers can be generated without algebraic computation. The entire divisor structure arises from a simple geometric rule: uniform parallel extension combined with systematic increase of inter-point distances.

As a consequence, the framework generates not only divisibility relations but also their global organization, including the hierarchy of composite numbers and the isolation of primes.

Primality as a positional phenomenon

Within the geometric framework, prime numbers are identified as positions that admit no nontrivial structural intersections. This reformulation shows that primality is fundamentally a positional property of the integers, determined by the global arrangement of the structure rather than by local arithmetic checks.

In particular, the infinite continuation of the construction guarantees the persistence of admissible isolated positions, yielding the positions of prime numbers along the infinite integer line.

Infinite structure from finite rules

Another significant aspect is that an infinite arithmetic structure is generated from a finite geometric rule. The Parascan–Margoș construction requires no limiting processes and no infinite computations; infinitude arises solely from iteration of a local distanciation principle.

This illustrates how infinite arithmetic phenomena can be encoded in finite geometric data.

Structural unification of arithmetic properties

The framework unifies several classical notions:

- divisibility as alignment,
- factorization as multiple alignment,
- primality as isolation,
- infinitude as structural persistence.

These notions are not introduced separately but arise naturally from a single geometric construction.

Conceptual significance

The approach highlights a fundamental duality of arithmetic: every integer is simultaneously a numerical quantity and a geometric position. The arithmetic properties of numbers are thus seen as manifestations of an underlying geometric organization.

This perspective does not alter classical results but clarifies their structural origin, providing a coherent geometric background for the arithmetic of the integers.

Final Remarks

The geometric framework developed here shows that the divisibility structure of the natural numbers can be generated, organized, and extended purely through positional relations. Primality emerges as a global geometric property, and the infinite sequence of prime positions is determined structurally rather than computationally.

While fully equivalent to classical arithmetic, this reformulation emphasizes geometry, invariance, and structure, offering a unified and conceptually transparent view of the natural numbers.

Final statement (Annals style)

The arithmetic of the natural numbers admits a complete and finite structural reformulation in which divisibility, factorization, and primality arise from geometric alignment within a distanciation-generated configuration. This reformulation preserves all classical properties while eliminating algebraic computation from the foundational level.

A BOX FOR PRIMALITY CONSTRUCTED EXCLUSIVELY GEOMETRICLY, BUT WHICH CAN BE CONSTRUCTED IDENTICALLY BY ALGEBRAIC CALCULATIONS.

1	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0	0	0
1	1	1	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	1	0	0	0	0	0	0
1	1	0	1	0	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	1	0	0	0	0
1	1	0	0	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	0	0	0	0	1	0	0
1	1	1	1	0	1	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0	0	0	0	1

A. Binary Matrix

n\d	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	0	1	0	2	0	3	0	4	0	5	0	6	0	7	0	8	0	9	0	10
3	0	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6	0	0
4	0	0	0	1	0	0	0	2	0	0	0	3	0	0	0	4	0	0	0	5
5	0	0	0	0	1	0	0	0	0	2	0	0	0	0	3	0	0	0	0	4
6	0	0	0	0	0	1	0	0	0	0	0	2	0	0	0	0	0	3	0	0
7	0	0	0	0	0	0	1	0	0	0	0	0	0	2	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	2	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	2	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

B. Decimal Matrix